

Handwritten HW 27

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28. Let A be an $n \times n$ real matrix with the property that $A^T = A$. Show that if $A\mathbf{x} = \lambda\mathbf{x}$ for some nonzero vector \mathbf{x} in \mathbb{C}^n , then, in fact, λ is real and the real part of \mathbf{x} is an eigenvector of A . [*Hint:* Compute $\bar{\mathbf{x}}^T A\mathbf{x}$, and use Exercise 27. Also, examine the real and imaginary parts of $A\mathbf{x}$.]

Solution:

Exercise 27 says, “Let A be an $n \times n$ real matrix with the property that $A^T = A$, let \mathbf{x} be any vector in \mathbb{C}^n , and let $q = \bar{\mathbf{x}}^T A\mathbf{x}$. The equalities below show that q is a real number by verifying that $\bar{q} = q$.”